

Question Paper

Quantitative Methods – II (132) : July 2003

Section A : Basic Concepts (40 Points)

- This section consists of questions with serial number 1 - 40.
- Answer all questions.
- Each question carries one point.

1. In which of the following conditions is the maximax criterion of decision making applicable?
 - I. The decision is made under conditions of certainty.
 - II. The decision is made under conditions of uncertainty.
 - III. The decision maker is perfectly pessimistic.
 - IV. The decision maker is perfectly optimistic.
 - V. The decision maker is neither perfectly optimistic nor perfectly pessimistic.
 - a. (I) only
 - b. (II) only
 - c. Both (II) and (III)
 - d. Both (II) and (IV)
 - e. Both (I) and (V).
2. Which of the following statements is/are **true** when decisions are made under conditions of risk?
 - I. All possible states of nature are not known.
 - II. All possible states of nature are known.
 - III. The decision maker does not have any knowledge of the probabilities of occurrence of the various states of nature.
 - IV. The decision maker has sufficient knowledge to assign probabilities of occurrence to the various states of nature.
 - a. (I) only
 - b. (II) only
 - c. Both (II) and (III)
 - d. Both (II) and (IV)
 - e. (III) only.
3. The expected value of perfect information is equal to the difference between
 - I. The highest gain possible under certain conditions.
 - II. The least gain possible under certain conditions.
 - III. The highest expected profit under uncertain conditions.
 - IV. The least expected profit under uncertain conditions.
 - V. The expected profit with perfect information.
 - VI. The highest expected profit without perfect information.
 - VII. The least expected profit without perfect information.
 - a. (I) and (II)
 - b. (III) and (IV)
 - c. (V) and (VI)
 - d. (V) and (VII)
 - e. (VI) and (VII).
4. The expected value of the number of successes of a binomial distribution is equal to
 - a. Zero
 - b. The number of trials
 - c. The probability of success in any trial
 - d. The probability of failure of any trial
 - e. The product of the number of trials and probability of success in any trial.

5. In a binomial distribution the probability of getting zero or more number of successes is equal to
 - a. 0
 - b. The probability of getting zero success
 - c. The probability of getting successes in all the trials
 - d. 1 minus the probability of getting successes in all the trials
 - e. 1.
6. Given a probability distribution for a random variable, the covariance of the variable with itself is equal to
 - a. 0
 - b. 1
 - c. The variance of the variable
 - d. The standard deviation of the variable
 - e. The expected value of the variable.
7. If x_1 and x_2 are two random variables independent of each other then, the variance of $x_1 + x_2$ is equal to the
 - a. Standard deviation of x_1 plus standard deviation of x_2
 - b. Standard deviation of x_1 minus standard deviation of x_2
 - c. Variance of x_1 plus variance of x_2
 - d. Variance of x_1 minus variance of x_2
 - e. Covariance of x_1 and x_2 .
8. If two random variables are independent of each other then the coefficient of correlation between them is equal to
 - a. 0
 - b. 1
 - c. Between 0 and 1
 - d. Between -1 and 0
 - e. -1.
9. Which of the following statements is **false** with regard to a normal distribution?
 - a. It is a discrete distribution
 - b. It is unimodal
 - c. It is a symmetrical distribution
 - d. The mean, median and mode are equal
 - e. None of the above.
10. The variance of the standard normal distribution is
 - a. Equal to its mean
 - b. Less than its mean
 - c. Equal to 0
 - d. Equal to its standard deviation
 - e. Greater than its standard deviation.
11. If a random variable X has a lognormal distribution then which of the following statements is **true**?
 - a. X is normally distributed
 - b. $\ln(X)$ is normally distributed
 - c. The expected value of X is zero
 - d. The expected value of X is 1
 - e. None of the above.
12. Which of the following statements is **false** with regard to sampling?
 - a. It is useful when the population is too large to be studied in full
 - b. It costs significantly less than a census
 - c. It is not suitable in case of destructive testing
 - d. It involves less work on statistical data processing than census
 - e. None of the above.

13. In which technique of random sampling the first element of the sample is randomly selected from the population and the remaining elements are selected at uniform intervals that are measured in time, order or space?
- Simple random sampling
 - Stratified sampling
 - Cluster sampling
 - Systematic sampling
 - None of the above.
14. Which of the following is a measure of the sampling error due to chance, with regard to a sample statistic?
- Standard deviation of the sample
 - Coefficient of variation of the sample
 - Variance of the sample
 - Variance of the population
 - Standard error of the sample statistic.
15. The sampling distribution of mean is
- A distribution of the observations included in a large sample
 - A distribution of the observations included in a small sample
 - A distribution of sample means of all possible samples of a specific size, that can be taken from a population
 - A distribution of sample means of all possible samples of a specific size that can be taken from different populations
 - A distribution of sample means of samples of different sizes taken from different populations.
16. How will the standard error of mean be affected if the sample size is reduced?
- It will decrease
 - It will remain unchanged
 - It will increase
 - It will change unpredictably
 - It will be equal to the population variance.
17. Which of the following sampling methods is suitable when the population can be divided into groups according to certain characteristics, so that each group is more or less homogeneous and there is a large variation among the groups?
- Stratified sampling
 - Cluster sampling
 - Simple random sampling
 - Systematic sampling
 - Judgmental sampling.
18. In which of the following conditions is a type I error said to have occurred?
- The null hypothesis is false and accepted
 - The null hypothesis is true and rejected
 - The null hypothesis is false and rejected
 - The null hypothesis is true and accepted
 - None of the above.
19. Which of the following is **true** with regard to t-distribution?
- It is multimodal
 - It is negatively skewed
 - It is positively skewed
 - There is a different t-distribution for different degrees of freedom
 - None of the above.

20. In which of the following conditions the finite population correction factor **need not** be used for calculating the standard error of a statistic?
- The population is finite.
 - The sampling is done without replacement.
 - The sampling is done with replacement.
- (I) only
 - (II) only
 - (III) only
 - Both (I) and (II)
 - Both (I) and (III).
21. In a left tailed test of hypothesis the
- Acceptance region starts from the left tail and extends to the right
 - Rejection region starts from the left tail and extends to the right upto a point determined by the significance level of the test
 - Entire region to the right of mean is the acceptance region
 - Entire region to the left of mean is the acceptance region
 - Entire region to the left of mean is the rejection region.
22. The means of two populations are represented by μ_1 and μ_2 . It has to be tested whether μ_1 is 5 less than μ_2 . Which of the following is the correct representation of the alternative hypothesis for such a test?
- $H_1 : \mu_1 = \mu_2 - 5$
 - $H_1 : \mu_1 \neq \mu_2 - 5$
 - $H_1 : \mu_1 - 5 = \mu_2$
 - $H_1 : \mu_1 - 5 \neq \mu_2$
 - $H_1 : \mu_1 + \mu_2 = 5$.
23. For samples of size greater than 1, the finite population correction factor
- Is always equal to 1
 - Is always more than 1
 - Is always constant irrespective of the sample size
 - Is always equal to 0
 - Is always less than 1.
24. If two variables **X** and **Y** are perfectly positively correlated then their covariance will be equal to
- 0
 - 1
 - The product of the standard deviations of **X** and **Y**
 - The sum of the standard deviations of **X** and **Y**
 - The difference between the standard deviations of **X** and **Y**.
25. If the regression coefficient of the independent variable in a simple regression equation is negative then, which of the following statements is **correct**?
- The coefficient of correlation between the variables is zero
 - The coefficient of correlation between the variables is the positive square root of the coefficient of determination
 - The coefficient of correlation between the variables is the negative square root of the coefficient of determination
 - The coefficient of correlation between the variables is 1
 - None of the above.

26. The proportion of variation in the dependent variable that is not explained by the regression line is equal to
- The coefficient of correlation
 - The coefficient of determination
 - 1 minus coefficient of correlation
 - 1 minus coefficient of determination
 - The coefficient of variation of the dependent variable, expressed as a proportion.
27. Which of the following is **false** with regard to hypothesis testing?
- The purpose of hypothesis testing is to find out whether there is a significant difference between the sample statistic and the hypothesized population parameter
 - Rejection of null hypothesis implies the acceptance of alternative hypothesis
 - The acceptance of null hypothesis proves that it is true
 - The rejection of null hypothesis does not prove that it is false
 - None of the above.
28. Which of the following is an example of a continuous random variable?
- The volume of liquid present in a container
 - The number of patients admitted to a hospital in a day
 - The number of defective units of a product in a batch of production
 - The number of spectators in a cinema hall
 - The number of professors in a college.
29. The number of possible outcomes in a single trial of a Bernoulli experiment is
- 0
 - 1
 - 2
 - Variable
 - Infinite.
30. Which of the following statements is **false**?
- An interval estimate is a range of values within which the actual value of the population parameter will always fall
 - An interval estimate indicates the probability that the true population parameter will fall within the given range of values
 - A point estimate may be either correct or incorrect
 - A point estimate is a single number that is used to estimate an unknown population parameter
 - None of the above.
31. If the standard error of estimate for a regression equation is zero then,
- The estimated value of the dependent variable will always be less than its observed value
 - The estimated value of the dependent variable will always be more than its observed value
 - The estimated value of the dependent variable will always be equal to its observed value
 - The coefficient of correlation between the variables involved will be zero
 - The coefficient of correlation between the variables involved will be positive.
32. Which of the following is **true** with regard to multicollinearity in multiple regression analysis?
- It increases the influence of the individual variables in the model
 - It reduces the predictive power of the model
 - It increases the reliability of the regression coefficients
 - It reduces the effectiveness of sensitivity analysis of the model
 - None of the above.

33. Which of the following patterns of $\bar{X}$ charts indicates that two distinct populations are being observed?
- Increasing trend
 - Jumps in process levels
 - Cycles in process level
 - Hugging the control limits
 - Hugging the centerline.
34. Which of the following components of a time series is expressed as a percentage of the estimated values of the dependent variable as per the secular trend equation?
- Irregular variation
 - Seasonal variation
 - Cyclical variation
 - Short term variation
 - None of the above.
35. Which of the following price indices has an upward bias?
- Laspeyres prices index
 - Paasche's price index
 - Fisher's ideal price index
 - Marshall-Edgeworth price index
 - None of the above.
36. Which of the following statements is **false**?
- An unweighted aggregates price index does not attach any importance to the consumption or usage levels of the items covered
 - Laspeyres price index is an unweighted aggregates price index
 - Paasches price index is a weighted aggregates price index
 - A price index reflects the overall change in price for the basket of goods considered
 - None of the above.
37. Which of the following is a technique for studying seasonal variations?
- Regression analysis
 - Residual method
 - Hypothesis testing
 - Ratio to moving average method
 - None of the above.
38. Which of the following statements is **not true** with regard to a value index?
- It measures changes in net monetary worth
 - It combines price changes and quantity changes
 - It is not as useful as price indices or quantity indices
 - It cannot distinguish between the price changes and quantity changes
 - An increase in the value index indicates with certainty that the prices have increased.
39. Which of the following statements is **not true** with regard to the coefficient of multiple correlation $R_{1,23}$ between the dependent variable, Y , and the independent variables, X_1 and X_2 ?
- It is a non-negative real number
 - It can take values between 0 and 1
 - It is dependent on the coefficient of correlation between Y and X_1
 - It is dependent on the coefficient of correlation between Y and X_2
 - It is independent of the coefficient of correlation between X_1 and X_2 .

40. Which of the following distributions is most suitable for testing the equality of means of more than two populations?
- a. Chi-square distribution
 - b. F distribution
 - c. Normal distribution
 - d. t distribution
 - e. None of the above.

END OF PART A

Section B : Problems (60 Points)

- This section consists of questions with serial number 41 - 70.
- Answer all questions.
- Points are indicated against each question.

41. For generating an $\bar{x}$ chart, 20 samples are taken. The sum of all the observations in the samples is 360 and the value to be used for the centre line is 1.5. The size of each sample is

- a. 5
- b. 10
- c. 12
- d. 20
- e. 36.

(1 point)

42. Given below are the probabilities associated with the possible prices of a share in future, in the secondary market:

Possible Prices (Rs.)	25	30	35	40
Probability (%)	20	40	30	10

The expected price of the share is

- a. Rs.27.5
- b. Rs.28.5
- c. Rs.31.5
- d. Rs.32.5
- e. Rs.37.5.

(1 point)

43. If the sample size is 100 and the proportion of defective pieces in a batch of output is 0.10 then, the upper control limit of **p chart** will be

- a. 0.01
- b. 0.03
- c. 0.09
- d. 0.10
- e. 0.19.

(1 point)

44. In a test of ANOVA, the following are found:

Estimated population variance based on the variance among sample means = 21.60

F ratio = 1.50

What is the estimated population variance based on the variance within the samples?

- a. 12.25
- b. 14.40
- c. 24.25
- d. 26.75
- e. 31.88.

(1 point)

45. A multiple regression relationship contains two independent variables. The standard error of estimate is 2.85. $\sum(Y - \hat{Y})^2 = 129.96$. The number of observations is

a. 11
b. 14
c. 16
d. 19
e. 20.

(1 point)

46. Two variables, X and Y, are related by a simple regression equation such that X is the independent variable and Y is the dependent variable. The following details are available:

Coefficient of correlation between X and Y = -0.80
Standard deviation of X = 16.4
Standard deviation of Y = 24.6

The slope of the regression equation is

a. -1.20
b. 1.20
c. 10.37
d. 16.20
e. 25.31.

(1 point)

47. The following data pertain to three commodities:

Commodity	Price in 2002 (Rs./kg)	Price in 1997 (Rs./kg)
Rice	12.50	10.50
Wheat	13.50	8.50
Pulses	25	20

The base year is 1997. The unweighted aggregates price index for the year 2002 is approximately

a. 115.62
b. 125.45
c. 130.77
d. 250.55
e. 410.00.

(1 point)

48. The following index values pertain to a year:

Fisher's ideal price index = 124.50
Laspeyres price index = 122.00

Paasche's price index for the year is approximately

a. 102.90
b. 115.50
c. 120.50
d. 127.05
e. 129.10.

(1 point)

49. The probability of failure in a Bernoulli experiment is 0.60. The experiment is repeated 25 times. The mean of the binomial distribution of the number of successes is

- a. 10
- b. 20
- c. 30
- d. 40
- e. 50.

(1 point)

50. The slope and the intercept on y-axis for a simple regression line are 3 and 5 respectively. If the independent variable has a value of 10, then the estimated value of the dependent variable is

- a. 3
- b. 5
- c. 20
- d. 35
- e. 50.

(1 point)

51. A variable X, which has a normal distribution, has a mean of 24 and a standard deviation of 5. The probability that the value of X lies in the interval 20 to 28 is approximately

- a. 24.7%
- b. 31.5%
- c. 38.5%
- d. 49.5%
- e. 57.6%.

(1 point)

Questions 52 and 53 relate to the following information:

52. United Enterprises Ltd. has its manufacturing units in the four regions of the country viz. North, South, East and West. The company is planning to change its performance appraisal method. The management of the company has proposed a new method to replace the existing method of performance appraisal. The HR department of the company has conducted a survey of the attitudes of the company employees working at the company's manufacturing units in the four regions, toward the performance appraisal methods. The following data were collected:

	East	West	North	South	Total
Number of employees who prefer the present method	60	70	50	70	250
Number of employees who prefer the proposed method	33	49	35	33	150
Total employees sampled	93	119	85	103	400

What is the chi-square statistic for the sample of observations?

- a. 0.0605
- b. 3.516
- c. 9.766
- d. 2.6485
- e. 0.8192.

(4 points)

53. At a 0.05 level of significance which of the following conclusions can be drawn?
- The expected frequencies of employees who prefer the present method of appraisal is same in all the four regions
 - The observed frequencies of employees who prefer the present method of appraisal is same in all the four regions
 - The expected frequencies of employees who prefer the proposed method of appraisal is same in all the four regions
 - There is no difference in attitudes toward the performance appraisal methods among the employees in the four regions
 - None of the above.

(1 point)

54. Sixty percent of the medium sized software firms based in the northern region of the country planned to send their representatives to Europe in order to explore business opportunities there. An export-import group in Manchester, U.K., has invited 16 medium sized software firms based in the northern region of the country, to participate in a conference to explore the possibilities of developing trading relationships.

What is the probability that at least 10 of the invited software firms will send their representatives to the conference?

- 19.83%
- 31.05%
- 32.88%
- 52.71%
- 47.29%.

(2 points)

55. A wine producer is contemplating an expansion in its sales territory upto the southern part of the state. In order to do so, it must build a new brewery in that part of the state so that it can avoid the problems related with transportation. The producer has to make a decision on the capacity of the brewery that should be constructed. Three capacities of the brewery are under consideration viz, Small, Medium and Large. The management of the firm has advised that the capacity should be based on the projected net profit for the third year of operation of each of the three capacities under consideration. The marketing department of the firm has made an assessment of the future demand level during the third year of operation of the brewery, and according to it the sales during the third year of operation may grow by 5 percent, 10 percent or 15 percent. The projected net profits from the three proposed brewery capacities in the third year of operation under the three types of growth in demand, are given below:

Capacity	Net profit (Rs. millions)		
	Growth in Demand		
	5%	10%	15%
Small	75	110	125
Medium	55	100	140
Large	45	75	155

If the regret criterion for decision-making is used then what should be the brewery capacity?

- Small
- Medium
- Large
- Medium or large
- Cannot be determined.

(2 points)

56. A sample of paired observations of two random variables, X and Y, is given below:

X	3	6	2	4	5	4	7	1
Y	25	32	16	28	30	13	30	10

What is the covariance between X and Y?

- a. 10.12
- b. 11.25
- c. 13.86
- d. 14.35
- e. 16.48.

(2 points)

57. The Western Delight Restaurant prepares a chicken dish, Chicken Fun, every Sunday. The manager of the restaurant, wants to ensure that the restaurant earns the maximum profit on this dish. From each whole chicken that is cooked, two portions of Chicken Fun are prepared. The selling price per portion of the dish is Rs.50, which is considered to be a bargain price, and this has made the dish very popular. Each portion of the dish costs Rs.32 including labour and other costs of preparation. It is observed over a long period of time that the demand for the dish is normally distributed with a mean of 210 portions and a standard deviation of 50 portions. The dish is perishable and the unsold portions of the dish cannot be preserved and sold later.

How many whole chickens should be ordered by the manager of the restaurant?

- a. 88
- b. 90
- c. 94
- d. 95
- e. 96.

(2 points)

58. City Finance Ltd. offers various types of financing to its clients. Its clientele includes individuals as well as business firms. It offers leasing and hire purchase financing as well as direct loan to the business firms for a variety of asset purchases. Presently, there are 160 business firms among its clients; the remaining are individuals. It has been observed that the borrowers belonging to the business firm category tend to pay their repayment dues regularly. However, of late some irregularities have been observed in the payment habits of its individual borrowers. The individuals constitute 90 percent of all its borrowers. The managing director of the company ordered a random survey of all the accounts in the individual borrowers category. 144 accounts in the individual borrowers category were randomly surveyed and it was found that 108 of them were in excellent standing. An interval estimate for the absolute number of accounts belonging to the individual borrowers category that meet the requirements of excellent standing, has been constructed with a confidence level of 90 percent.

Which of the following represents the interval estimate for the absolute number of accounts in the individual borrowers category, which are in excellent standing?

- a. 108 ± 85.24
- b. 108 ± 80.78
- c. 144 ± 80.78
- d. 1080 ± 85.24
- e. 1080 ± 80.78 .

(4 points)

59. Rising prices of prescription drugs has been a cause of concern to many public interest groups, especially to those who are working for the welfare of senior citizens. One such group is planning to send their representation to the government on this issue with a request to pass suitable legislation that would force the pharmaceutical firms to offer the prescription drugs to senior citizens at a discounted price. The group has been collecting data on the prescription prices of many major drugs that include cardiological drugs, neurological drugs, gynaecological drugs etc. Recently it has collected a small sample of the prescription prices of a drug used to treat cardiac patients. The sample data collected are given below:

Price (in Rs.) :	125	115	130	120	118	107	106	112	114	113
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It is assumed that the prescription prices for this drug at all the retail outlets in the country are normally distributed.

Which of the following is the interval estimate for the true mean price (in Rs.) of the drug for the entire population, at a 90 percent confidence level?

- a. 116 ± 2.376
- b. 116 ± 4.355
- c. 116 ± 7.513
- d. 116 ± 13.771
- e. 116 ± 56.445 .

(4 points)

60. A placement consultant has been recruiting freshers from colleges in various types of companies. The placement consultant has observed that the mean annual salary offered to the candidates placed by him in the retailing firms was Rs.48000 in 2002. From January 2003 to March, 2003, the consultant has placed as many as 150 fresh graduates in the retailing firms. The consultant is of the opinion that the mean annual salary offered by the retailing firms to the candidates placed by him from January 2003 to March, 2003, is more than the mean salary offered by the same type of employers in 2002. He has collected a sample of 36 candidates placed in 2003 in the retailing firms and has observed that the mean annual salary offered to them is Rs.50,400 with a standard deviation of Rs.1,800. It is to be determined whether the candidates placed by the consultant from January 2003 to March, 2003, in the retailing firms, have been offered mean annual salary greater than the mean annual salary offered to the candidates in 2002.

Which of the following statements are **correct**?

- I. A two tailed test of hypothesis should be done.
 - II. A left tailed test of hypothesis should be done.
 - III. A right tailed test of hypothesis should be done.
 - IV. The standard error of mean is estimated to be Rs.300.
 - V. The standard error of mean is estimated to be Rs.262.41.
 - VI. The normal distribution cannot be used.
 - VII. The normal distribution can be used.
 - VIII. The standardized value of the sample mean is 8.00.
 - IX. The standardized value of the sample mean is 9.15.
 - X. At a significance level of 5 percent the critical value for the test is 1.64.
 - XI. At a significance level of 5 percent the critical value for the test is -1.64.
 - XII. The rejection area lies in the left tail of the sampling distribution.
 - XIII. The rejection area lies in both the tails of the sampling distribution.
 - XIV. The rejection area lies in the right tail of the sampling distribution.
 - XV. At a significance level of 5 percent, it can be concluded that the mean annual salary offered by the retailing firms to the candidates recruited during January, 2003 to March, 2003 is equal to Rs.48,000.
 - XVI. At a significance level of 5 percent, it can be concluded that the mean annual salary offered by the retailing firms to the candidates recruited during January, 2003 up to March, 2003 is greater than Rs.48000.
- a. (I), (V), (VI) and (VIII)
 - b. (II), (V), (VI) and (IX)
 - c. (III), (IV), (VI), (VIII), (XI) and (XII)
 - d. (III), (V), (VII), (IX), (X), (XIV) and (XVI)
 - e. (II), (IV), (VI), (VIII), (XIII) and (XV).

(3 points)

61. Southern Garments, a local manufacturer of women's clothing, markets its products in Hyderabad and adjoining places. From its past records it knows that its products are carried in 25 percent of the women's clothing stores in Hyderabad. The marketing manager of the firm wants to know if the percentage of women's clothing stores in Secunderabad, which carry its products is less than the percentage in Hyderabad. He conducted a sample survey of the women's clothing stores in Secunderabad. It was found out that the ratio of the number of clothing stores in Secunderabad which did not carry its products, to the number of clothing stores in Secunderabad which carried its products, was 4:1. Eight out of the women's clothing stores in Secunderabad which were sampled, carried its products.

Which of the following statements are **correct**?

- I. A right tailed test of hypothesis is appropriate.
 - II. A two tailed test of hypothesis is appropriate.
 - III. A left tailed test of hypothesis is appropriate.
 - IV. The alternative hypothesis is $p = 0.25$.
 - V. The alternative hypothesis is $p > 0.25$.
 - VI. The alternative hypothesis is $p < 0.25$.
 - VII. The standard error of proportion is 0.06325.
 - VIII. The standard error of proportion is 0.25.
 - IX. The standard error of proportion is 0.3333.
 - X. The standard error of proportion is 0.06847.
 - XI. The rejection region falls under the right tail.
 - XII. The rejection region falls in the middle of the sampling distribution.
 - XIII. The rejection region falls under the left tail.
 - XIV. The critical value for the test is 1.28 at a significance level of 10%.
 - XV. The critical value for the test is -1.28 at a significance level of 10%.
 - XVI. The standardized value of the sample proportion is -0.791.
 - XVII. The standardized value of the sample proportion is -0.73.
 - XVIII. At a significance level of 10%, it can be concluded that the percentage of women's clothing stores in Secunderabad which carry the firm's products is significantly less than the percentage in Hyderabad.
 - XIX. At a significance level of 10% it can be concluded that the percentage of women's clothing stores in Secunderabad, which carry the firm's products is not significantly less than the percentage in Hyderabad.
- a. (I), (IV), (VII), (XIV), (XVI) and (XVIII)
 - b. (II), (IV), (VIII), (XII), (XIV), (XVI) and (XVIII)
 - c. (II), (V), (IX), (XI), (XIV) and (XVI)
 - d. (III), (VI), (X), (XIII), (XV), (XVII) and (XIX)
 - e. (I), (VI), (VIII), (XI), (XVI) and (XVIII).

(3 points)

62. There is a population with known variance. If a sample of size **m** is taken from this population then the standard error of mean will be 2.25. If a sample of size **n** is taken from this population then the standard error of mean will be 1.5.

What will be the standard error of mean if a sample of size **m+n** is taken from this population?

- a. 1.50
- b. 2.167
- c. 1.444
- d. 0.692
- e. 1.248.

(2 points)

63. Two medical research laboratories, Alpha Pharma Ltd (APL) and Orient Laboratories Ltd (OLL), have been independently conducting research for producing drugs that will provide relief to those who are suffering from asthma. Very recently they have produced drugs for providing relief to the asthma patients. The drug produced by APL was tested on a group of 80 asthma patients and it provided an average of 10.5 hours of relief, with a standard deviation of 1.5 hours. The drug produced by OLL was tested on a group of 64 asthma patients and it provided an average of 9.6 hours of relief, with a standard deviation of 2.5 hours. It is to be tested whether the mean hours of relief provided by the drug produced by APL is more than the mean hours of relief provided by the drug produced by OLL.

Which of the following statements are **correct**?

- I. A two tailed test of hypothesis on difference between means is appropriate.
 - II. A one tailed test of hypothesis on difference between means is appropriate.
 - III. The estimated standard error of difference between the means is 0.1677 hours.
 - IV. The estimated standard error of difference between the means is 0.3125 hours.
 - V. The estimated standard error of difference between the means is 0.3547 hours.
 - VI. The normal distribution is not applicable for the test of hypothesis.
 - VII. The normal distribution is applicable for the test of hypothesis.
 - VIII. The standardized difference between the sample means is 5.37.
 - IX. The standardized difference between the sample means is 2.88.
 - X. The standardized difference between the sample means is 2.54.
 - XI. At a significance level of 4 percent it can be concluded that the mean hours of relief provided by the drug produced by APL is significantly less than the mean hours of relief provided by the drug produced by OLL.
 - XII. At a significance level of 4 percent it can be concluded that there is no significant difference between the mean hours of relief provided by the drugs produced by APL and OLL.
 - XIII. At a significance level of 4 percent it can be concluded that the mean hours of relief provided by the drug produced by APL is more than the mean hours of relief provided by the drug produced by OLL.
- a. (I), (III), (VI), (VIII) and (XI)
 - b. (V), (VI), (IX), and (XII)
 - c. (IV), (VI), (VIII) and (XI)
 - d. (II), (V), (VII), (X) and (XIII)
 - e. (III), (VI), (VIII) and (XII).

(3 points)

64. Prestige Industries Ltd. (PIL) has a sales force consisting of 150 salespersons having various number of years of sales experience. The sales manager of the company wants to establish a relationship between the number of years of experience and annual sales. The following data are collected by him from a random sample of ten salespersons of the company:

Number of years of experience	Annual Sales (in Rs.000s)
2	78
11	121
4	95
11	117
5	90
9	109
5	100
7	101
14	134
12	115

A simple regression relationship has to be developed for estimating the annual sales on the basis of the number of years of experience of the salesperson.

Which of the following is the estimated amount of annual sales if a salesperson has 10 years of experience?

- a. Rs. 40,000
- b. Rs. 74,000
- c. Rs. 78,000
- d. Rs.1,14,000
- e. Rs. 34,000.

(3 points)

65. The following data shows the sales and operating costs of a firm over a period of eight years:

Year	1992	1993	1994	1995	1996	1997	1998	1999
Sales (Rs in lakhs)	106	140	144	155	136	150	146	143
Operating costs (Rs in lakhs)	100	114	120	125	105	124	122	118

What is the coefficient of correlation between sales and operating costs of the firm?

- a. 0.676
- b. 0.374
- c. 0.553
- d. 0.854
- e. 0.904.

(3 points)

66. A simple regression relationship was developed between the variables X and Y, with X as the independent variable. The estimated value of Y when X = 2, is 7.0; and the estimated value of Y when X = 8, is 11.8.

$$\Sigma Y^2 = 791.31 \quad \Sigma XY = 452.7 \quad \Sigma Y = 78.1 \quad n = 8$$

The standard error of estimate for the regression line is

- a. 1.1113
- b. 8.4572
- c. 7.8482
- d. 7.5123
- e. 6.5893.

(3 points)

67. A simple regression relationship was developed between two variables X and Y, with X as the independent variable.

$$\Sigma Y = 792 \quad \Sigma Y^2 = 79144 \quad \Sigma (Y - \hat{Y})^2 = 200 \quad n = 8$$

The coefficient of determination is

- a. 0.2717
- b. 0.7283
- c. 0.3731
- d. 0.2137
- e. 0.7863.

(1 point)

68. Two simple regression relationships have been developed between variables X and Y; and between variables W and Y. In both the relationships variable Y is the dependent variable. The coefficient of correlation between variables X and Y is 0.85 and the coefficient of correlation between variables W and Y is -0.92.

Which of the following statements is/are **correct**?

- a. The proportion of variation in variable Y that is not explained by variable X is 0.1536
- b. The proportion of variation in variable Y that is not explained by variable W is 0.2775
- c. The proportion of variation in variable Y that is explained by variable X is 0.7225
- d. The proportion of variation in variable Y that is explained by variable W is more than the proportion of variation in variable Y that is explained by variable X
- e. Both (c) and (d) above.

(1 point)

69. Pyramid Networks Ltd. manufactures and sells network access software products. Its sales have been increasing in the recent years. The company is planning to expand its work force in response to the increasing trend shown by the sales of its products. The future manpower requirement is expected to be dependent on the level of future sales. The company requires an estimate for the sales in the year 2003. The following data are collected:

Year	Sales (Rs. in lakhs)
2002	160
2001	135
2000	105
1999	85
1998	60
1997	35

The sales for the year 2003, estimated on the basis of a linear trend equation, is

- a. Rs.150.16 lakh
- b. Rs.168.84 lakh
- c. Rs.175.34 lakh
- d. Rs.183.68 lakh
- e. Rs.195.25 lakh.

(2 points)

70. A linear trend equation was developed for relating the annual operating cost of Alpha industries Ltd., with the year. The operating cost for the year 1999 was observed to be Rs.63.18 lakh and the relative cyclical residual for that year was 8. The operating cost for the year 2000 was observed to be Rs.72.45 lakh and the relative cyclical residual for that year was 5. The operating cost for the year 2001 was observed to be Rs.77.91 lakh. In the linear trend equation, the time variable (**X**) has been coded as $x = X - 1998$.

What was the relative cyclical residual for the year 2001?

- a. 2
- b. -2
- c. 4
- d. 3
- e. -3.

(4 points)

END OF PART A

Suggested Answers

Quantitative Methods – II (132) : July 2003

Section A : Basic Concepts

1. Answer : (d)
Reason : The maximax criterion of decision making is applicable when the conditions are uncertain and the decision maker is perfectly optimistic. It is not applicable when the conditions are certain, or the decision maker is perfectly pessimistic, or the decision maker can not be strictly classified to be perfectly optimistic or perfectly pessimistic.
2. Answer : (d)
Reason : When decisions are made under conditions of risk all possible states of nature are known to the decision maker and the decision maker has sufficient knowledge to assign probability of occurrence to the various states of nature.
3. Answer : (c)
Reason : The expected value of perfect information is equal to the difference between the expected profit with perfect information and highest expected profit without perfect information.
4. Answer : (e)
Reason : The mean (expected value) of the number of successes in a binomial distribution is equal to the product of the number of trials and probability of success in any trial.
5. Answer : (e)
Reason : In a binomial distribution the probability of getting zero or more number of successes
$$= P(x = 0) + P(x = 1) + \dots P(x = n) = 1.00.$$
6. Answer : (c)
Reason : The covariance of any random variable with itself is equal to the variance of the variable.
7. Answer : (c)
Reason : $\text{Var}(x_1 + x_2) = \text{Var}(x_1) + \text{Var}(x_2) + 2\text{Cov}(x_1, x_2)$
If x_1 and x_2 are independent then, $\text{Cov}(x_1, x_2) = 0$
 $\therefore \text{Var}(x_1 + x_2) = \text{Var}(x_1) + \text{Var}(x_2).$
8. Answer : (a)
Reason : If two random variables are independent then their covariance = 0
$$\therefore \text{Coefficient of correlation} = \frac{\text{Covariance}}{\text{Product of the standard deviations}} = 0$$
9. Answer : (a)
Reason : Normal distribution is a continuous distribution. It is unimodal and symmetrical. The mean, median and mode are equal.
10. Answer : (d)
Reason : For a standard normal distribution $\sigma = 1$
$$\therefore \text{Variance} = \sigma^2 = 1^2 = 1 = \text{standard deviation}.$$
11. Answer : (b)
Reason : If a random variable x has a log normal distribution, then $\ln(x)$ will be normally distributed. The expected value of the variable need not necessarily be 0 or 1.
12. Answer : (c)
Reason : Sampling is the appropriate technique in case of destructive testing. All other statements are true with regard to sampling.

13. Answer : (d)
Reason : In systematic sampling the first item to be included in the sample is randomly selected from the population and the remaining items are selected at a uniform interval that is measured in time or space. The procedure involved in cluster sampling and stratified is different from this.
14. Answer : (e)
Reason : The standard error of a sample statistic is a measure of the sampling error due to chance.
15. Answer : (c)
Reason : The sampling distribution of mean is a distribution of sample means of all possible samples of a specific size, that can be taken from a population.
16. Answer : (c)
Reason : The standard error of mean will increase if sample size is reduced.
- $$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$
- If 'n' is reduced then $\sigma_{\bar{x}}$ will increase.
17. Answer : (a)
Reason : When the population can be divided into groups according to certain characteristics, so that each group is more or less homogeneous and there is a large variation among the groups, stratified sampling is the suitable method.
18. Answer : (b)
Reason : A type I error is said to have occurred if the null hypothesis is true and rejected.
19. Answer : (d)
Reason : t distribution is unimodal and symmetrical. However there is a different t distribution for different degrees of freedom.
20. Answer : (e)
Reason : The finite population correction factor need not be used for calculating the standard error of a statistic if the population is finite and sampling is done with replacement.
21. Answer : (b)
Reason : In a left tailed test of hypothesis the rejection region starts from the left tail and extends to the right upto a point (known as the critical value) determined by the significance level of the test.
22. Answer : (b)
Reason : Null hypothesis : $H_0 : \mu_1 = \mu_2 - 5$
Alternative hypothesis : $H_1 : \mu_1 \neq \mu_2 - 5$.
23. Answer : (e)
Reason : Finite population correction factor = $\sqrt{\frac{N-n}{N-1}}$
 $(N-n) < (N-1)$
 $\therefore$ Finite population correction factor is always less than 1 for $n > 1$.
24. Answer : (c)
Reason : $r = \frac{\text{Cov}(x, y)}{\sigma_x \sigma_y}$
 $r = +1 \Rightarrow \text{Cov}(x, y) = \sigma_x \cdot \sigma_y$.

25. Answer : (c)
Reason : $\hat{y} = a + bx$
'b' is the regression coefficient of the independent variable. $b < 0$ implies that the variables x and y are negatively correlated.
 $\therefore$ Coefficient of correlation = Negative square root of coefficient of determination.
26. Answer : (d)
Reason : The proportion of variation in the dependent variable that is not explained by the regression line
$$= 1 - \frac{\Sigma(\hat{y} - \bar{y})^2}{\Sigma(y - \bar{y})^2} = 1 - r^2 = 1 - \text{Coefficient of determination.}$$
27. Answer : (c)
Reason : The acceptance of null hypothesis does not prove that it is true because there are situations when the sample statistic falls within the acceptance region, though null hypothesis is false. That is why type II error occurs.
28. Answer : (a)
Reason : The volume of liquid present in a container is a continuous random variable.
29. Answer : (c)
Reason : The number of possible outcomes in a single trial of a Bernoulli experiment is 2.
30. Answer : (a)
Reason : An interval estimate is a range of values within which the actual value of the population parameter, may (or may not) fall.
31. Answer : (c)
Reason : The standard error of estimate for a regression equation is given by $\sqrt{\frac{\Sigma(y - \hat{y})^2}{n - 2}}$ where y is the actual value, $\hat{y}$ is the estimated value from the regression equation and n is the number of points. The above relation implies that the standard error of estimate will be zero, if the actual values and the estimated values are equal and in such case the correlation is either perfectly positive (+1) or perfectly negative (-1).
32. Answer : (d)
Reason : Multicollinearity reduces the effects of the individual variables in the model; however the variables may together explain the dependent variable. It does not reduce the predictive power of the model. It reduces the effectiveness of sensitivity analysis of the model. Hence (a), (b) and (c) are not true, (d) is true.
33. Answer : (d)
Reason : (a) indicates that the process mean is increasing. (b) indicates that the process mean may be shifting. (c) indicates that there may be random variations or there may be the influence of such factors which vary cyclically. (e) indicates that the process variability has been reduced to a great extent. (d) indicates that two distinct populations are being observed. Hence (d) is the answer.
34. Answer : (c)
Reason : (a), (b) and (d) are not expressed as percentage of the estimated secular trend values. Cyclical variations are expressed as a percentage of the estimated secular trend values, hence (c) is the answer.
35. Answer : (a)
Reason : Laspeyres price index has an upward bias. Paasches price index has a downward bias. Fisher's ideal price index is free from bias. Marshall-Edgeworth price index closely approximates Fisher's ideal price index, hence this can also be said to be nearly free from bias. Hence (a) is the answer.

36. Answer : (b)
Reason : Laspeyres price index is a weighted aggregates price index. Alternatives (a), (c) and (d) are true.
37. Answer : (d)
Reason : The technique used for studying seasonal variations is known as ratio to moving average method.
38. Answer : (e)
Reason : A value index can not distinguish between the price changes and quantity changes. Hence an increase in the value index cannot indicate with certainty that the prices have increased.
39. Answer : (e)
Reason : The coefficient of multiple correlation is dependent on the coefficients of correlation between Y and X_1 , between Y and X_2 , and between X_1 and X_2 . Alternatives (a), (b), (c) and (d) are true.
40. Answer : (b)
Reason : F-distribution have to be used for testing the equality of more than two population means, hence (b) is the answer. (a) is used for testing the equality of more than two population proportions. (c) and (d) can not be used for testing the equality of more than two population means.

Part B : Problems

41. Answer : (c)

Reason : Let the size of each sample be x .

The value to be used for the centre line is the grand mean. In this case the grand mean $(\bar{\bar{x}}) = 1.8$. Sum of all the observations $= \Sigma x = 360$ (given)

$$\text{Now, } \bar{\bar{x}} = \frac{\Sigma x}{n \times k}$$

Where,

n = Sample size = ?

k = number of samples = 20 (given)

$$\therefore n = \frac{\Sigma x}{\bar{\bar{x}} \cdot k}$$

$$\text{or } n = \frac{360}{20 \times 1.5} = 12$$

42. Answer : (c)

$$\begin{aligned} \text{Reason : The expected price} &= \Sigma (\text{Possible Price} \times \text{Probability}) \\ &= 25 \times 0.20 + 30 \times 0.40 + 35 \times 0.30 + 40 \times 0.10 \\ &= 31.5 \end{aligned}$$

43. Answer : (e)

$$\begin{aligned} \text{Reason : The lower control limit} &= p + 3 \sqrt{\frac{pq}{n}} = 0.1 + 3 \times \sqrt{\frac{0.1 \times 0.9}{100}} \\ &= 0.1 + 3 \times 0.03 = 0.10 + 0.09 = 0.19 \end{aligned}$$

44. Answer : (b)

Reason : The required estimate of population variance based on the variance within the sample will be

$$\frac{21.6}{1.5} = 14.40$$

45. Answer : (d)

Reason : The number of data points can be obtained as $\frac{\Sigma (y - \hat{y})^2}{s_e^2} + (k + 1)$

$$= \frac{129.96}{2.85 \times 2.85} + 2 + 1 = 19$$

46. Answer : (a)

Reason : The slope (b) of a regression equation is given by

$$b = \frac{\text{Cov}(x, y)}{\sigma_x^2} = r \frac{\sigma_y}{\sigma_x} = -0.80 \times \frac{24.6}{16.4} = -1.20$$

47. Answer : (c)

$$\begin{aligned} \text{Reason : Unweighted aggregates price index} &= \frac{\Sigma p_n}{\Sigma p_0} \times 100 \\ &= \frac{(12.50 + 13.50 + 25)}{(10.50 + 8.50 + 20)} \times 100 = 130.77 \end{aligned}$$

48. Answer : (d)

Reason : Fisher's ideal price index = $\sqrt{\text{Laspeyres price index} \times \text{Paasches price index}}$

$$\begin{aligned}\text{Paasches price index} &= \frac{(\text{Fisher's ideal price index})^2}{\text{Laspeyres price index}} \\ &= \frac{(124.5)^2}{122} = 127.05\end{aligned}$$

49. Answer : (a)

Reason : Mean of a binomial distribution = $n \cdot p$

$\therefore$ The mean of the binomial distribution of the number of successes is = $25(1-0.60) = 10$

50. Answer : (d)

Reason : If y is dependent on variable x then we can write that the estimated value of y , $\hat{y} = 3x + 5$.

When $x = 5$ the estimated value of y would be $3 \times 10 + 5 = 35$.

51. Answer : (e)

Reason : X lies within ± 0.8 times standard deviation $\left(\frac{28-24}{5} = 0.8 \text{ and } \frac{20-24}{5} = -0.8 \right)$ of mean. So

the probability is $2 \times 0.2881 = 0.5762$ i.e. 57.6%

52. Answer : (d)

Reason :

		East	West	North	South
A	Total number sampled	93	119	85	103
B	Estimated proportion who prefer the present method	0.625	0.625	0.625	0.625
C	Expected number of employees who prefer the present method = $A \times B$	58.125	74.375	53.125	64.375
D	Estimated proportion who prefer the proposed method	0.375	0.375	0.375	0.375
E	Expected number of employees who prefer the proposed method = $A \times D$	34.875	44.625	31.875	38.625

Estimated proportion of employees who prefer the present method

$$= \frac{60+70+50+70}{93+119+85+103} = \frac{250}{400} = 0.625$$

Estimated proportion of employees who prefer the proposed method

$$= 1 - 0.625 = 0.375$$

Observed and expected frequencies:

	East	West	North	South
Frequency of employees who prefer the present method:				
Observed (f_o)	60	70	50	70
Expected (f_e)	58.125	74.375	53.125	64.375
Frequency of employees who prefer the proposed method:				
Observed (f_o)	33	49	35	33
Expected (f_e)	34.875	44.625	31.875	38.625

$$\text{Chi-square statistic, } \chi^2 = \frac{\sum (f_o - f_e)^2}{f_e}$$

f_o	60	70	50	70	33	49	35	33	
f_e	58.125	74.375	53.125	64.375	34.875	44.625	31.875	38.625	
$f_o - f_e$	1.875	-4.375	-3.125	5.625	-1.875	4.375	3.125	-5.625	
$(f_o - f_e)^2$	3.516	19.141	9.766	31.641	3.516	19.141	9.766	31.641	
$\frac{(f_o - f_e)^2}{f_e}$	0.0605	0.2574	0.1838	0.4915	0.1008	0.4289	0.3064	0.8192	$\frac{\sum (f_o - f_e)^2}{f_e} = 2.6485$

53. Answer : (d)

Reason : H_0 : $P_E = P_W = P_N = P_S$

H_1 : P_E, P_W, P_N, P_S are not all equal

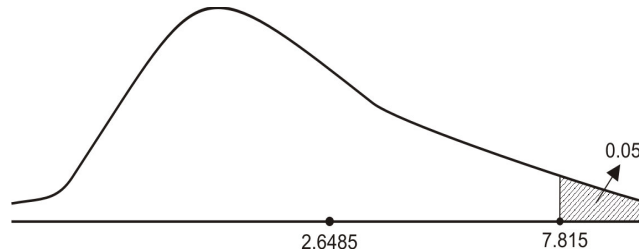
Degrees of freedom = (rows - 1) (columns - 1)

= (2 - 1) (4 - 1) = 3

Chi-square (df = 3, $\alpha = 0.05$) = 7.815 = critical value

Sample Chi-square statistic (2.6485) is less than the critical value (7.815) and it falls in the acceptance region.

Hence we accept the null hypothesis that there is no difference in attitudes toward the performance appraisal methods in the four regions in which the company's units are located.



54. Answer : (d)

Reason : $P(\text{at least } 10) = P(10) + P(11) + P(12) + P(13) + P(14) + P(15) + P(16)$

$$P(10) = {}^{16}C_{10} (0.60)^{10} (0.40)^{16-10} = 0.19833$$

$$P(11) = {}^{16}C_{11} (0.60)^{11} (0.40)^{16-11} = 0.16223$$

$$P(12) = {}^{16}C_{12} (0.60)^{12} (0.40)^{16-12} = 0.10142$$

$$P(13) = {}^{16}C_{13} (0.60)^{13} (0.40)^{16-13} = 0.04681$$

$$P(14) = {}^{16}C_{14} (0.60)^{14} (0.40)^{16-14} = 0.01505$$

$$P(15) = {}^{16}C_{15} (0.60)^{15} (0.40)^{16-15} = 0.00301$$

$$P(16) = {}^{16}C_{16} (0.60)^{16} (0.40)^{16-16} = 0.00028$$

$$P(10) + P(11) + P(12) + P(13) + P(14) + P(15) + P(16) = 0.52713 \text{ i.e. } 52.71\% \text{ (approx)}$$

55. Answer : (b)

Reason :

Net profit (Rs millions)

Capacity	Growth in demand			Maximum regret
	5%	10%	15%	
Small	75-75 = 0	110-110 = 0	155-125 = 30	30
Medium	75-55 = 20	110-100 = 10	155-140 = 15	20
Large	75-45 = 30	110-75 = 35	155-155 = 0	35

The decision should minimize the maximum regret. This happens if Medium capacity is selected. Minimum out of maximum regrets is Rs.20 millions.

56. Answer : (c)

$$\text{Reason : Cov (X,Y)} = \frac{\sum(X - \bar{X})(Y - \bar{Y})}{n-1}$$

A	X	3	6	2	4	5	4	7	1	$\sum X = 32$
B	Y	25	32	16	28	30	13	30	10	$\sum Y = 184$
C	$X - \bar{X}$	-1	2	-2	0	1	0	3	-3	
D	$Y - \bar{Y}$	2	9	-7	5	7	-10	7	-13	
E	$C \times D$	-2	18	14	0	7	0	21	39	$\sum(X - \bar{X})(Y - \bar{Y}) = 97$

$$\bar{X} = \frac{\sum X}{n} = \frac{32}{8} = 4$$

$$\bar{Y} = \frac{\sum Y}{n} = \frac{184}{8} = 23$$

$$\therefore \text{Cov (X,Y)} = \frac{97}{8-1} = 13.86 \text{ (approx).}$$

57. Answer : (e)

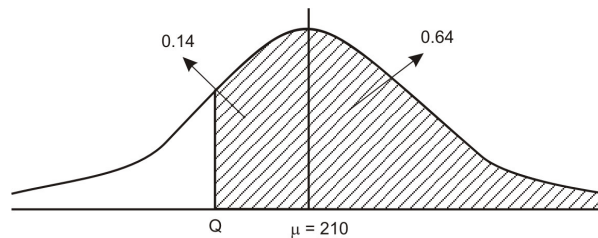
Reason : Minimum required probability of selling each portion of the dish,

$$p^* = \frac{ML}{MP + ML}$$

Marginal profit, MP = 50 - 32 = Rs.18

Marginal loss, ML = Rs.32

$$\therefore p^* = \frac{32}{32+18} = 0.64$$



The Z-value for the area 0.14 to the left of the mean is -0.36.

$$\therefore -0.36 = \frac{x-210}{50}$$

$$x = 210 - (0.36 \times 50) = 192$$

$\therefore$ The restaurant should prepare 192 portions at the optimal level.

$$\therefore \text{The number of chickens that should be ordered} = \frac{192}{2} = 96.$$

58. Answer : (e)

$$\text{Reason : Total number of individual borrower accounts} = \frac{160}{(1-0.90)} \times 0.90 = 1440$$

$$\text{Number of individual borrower accounts surveyed} = 144$$

$$\text{Sampling fraction} = \frac{n}{N} = \frac{144}{1440} = 0.10 > 0.05$$

Hence this can be treated as sampling done from a finite population.

$$\therefore \text{Proportion of excellent borrowers in the sample, } \bar{p} = \frac{108}{144} = 0.75$$

$$\bar{q} = 1 - \bar{p} = 1 - 0.75 = 0.25$$

$$\begin{aligned} \text{Estimated standard error of proportion, } \hat{\sigma}_{\bar{p}} &= \sqrt{\frac{\bar{p}\bar{q}}{n}} \sqrt{\frac{N-n}{N-1}} \\ &= \sqrt{\frac{0.75 \times 0.25}{144}} \sqrt{\frac{1440-144}{1440-1}} \\ &= 0.03608 \times 0.9490 \\ &= 0.0342 \end{aligned}$$

Since both $n\bar{p}$ and $n\bar{q}$ are greater than 5, the sample can be treated as a large sample and normal distribution can be used. For a confidence level of 90%, $Z = \pm 1.64$

$$1.64 \hat{\sigma}_{\bar{p}} = 1.64 \times 0.0342 = 0.0561$$

Confidence interval for the proportion of accounts in the individual borrower category who are in the excellent standing = 0.75 ± 0.0561

$\therefore$ Confidence interval for the absolute number of accounts in that category

$$= (0.75 \times 1440) \pm (0.0561 \times 1440)$$

$$= 1080 \pm 80.78$$

59. Answer : (b)

Reason : Sample mean price (in Rs.), $\bar{x} =$

$$\frac{125+115+130+120+118+107+106+112+114+113}{10} = 116$$

Sample variance $s^2 =$

$$\left[\frac{(125-116)^2 + (115-116)^2 + (130-116)^2 + (120-116)^2 + (107-116)^2 + (106-116)^2 + (112-116)^2 + (114-116)^2 + (113-116)^2}{10-1} \right] = \frac{508}{9}$$

$$\therefore \text{Sample standard deviation, } s = \sqrt{s^2} = \sqrt{\frac{508}{9}} = 7.513$$

$$\text{Estimated standard error of mean, } \hat{\sigma}_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{7.513}{\sqrt{10}} = 2.376$$

Since the sample is small (< 30) and the population standard deviation is not known, the t distribution is appropriate.

∴ For 90% confidence level and degrees of freedom = 10 – 1 = 9, t = 1.833.

$$\begin{aligned}\therefore \text{Confidence interval for the true mean price} &= 116 \pm (1.833 \times 2.376) \\ &= 116 \pm 4.355\end{aligned}$$

60. Answer : (d)

Reason : $H_0 : \mu = 48000$

$H_1 : \mu > 48000$

$$\text{Sampling fraction} = \frac{n}{N} = \frac{36}{150} = 0.24 > 0.05$$

Hence this should be treated as sampling from finite population.

$$\begin{aligned}\text{Standard error of mean, } \hat{\sigma}_{\bar{x}} &= \frac{S}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}} \\ &= \frac{1800}{\sqrt{36}} \sqrt{\frac{150-36}{150-1}} = 262.41\end{aligned}$$

Since the sample size is $36 > 30$ the normal distribution can be used,

$$\therefore \text{Standardized value of sample mean, } Z = \frac{\bar{x} - \mu}{\hat{\sigma}_{\bar{x}}} = \frac{50400 - 48000}{262.41} = 9.15$$

At the significance level of 5%, the critical value for the test = 1.64

Standardized value (9.15) > Critical value (1.64).

Hence the sample statistic falls in the rejection region. So the null hypothesis is rejected and alternative hypothesis is accepted.

Hence at 5% significance level it can be concluded that the mean annual salary offered to the candidates placed in the retail firms from January 2003 upto March, 2003, is more than Rs.48,000.

61. Answer : (d)

Reason : Proportion of women's clothing stores in Secunderabad which were sampled and found to carry the products of the firm, $\bar{p} = \frac{1}{4+1} = 0.20$

$$\text{Sample size} = \frac{8}{0.20} = 40$$

$H_0 : p = 0.25$

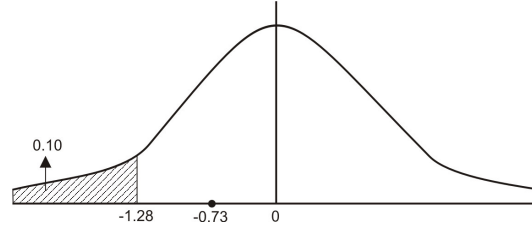
$H_1 : p < 0.25$

$$\begin{aligned}\text{Standard error of proportion, } \sigma_{\bar{p}} &= \sqrt{\frac{p_{H_0} q_{H_0}}{n}} \\ &= \sqrt{\frac{0.25(1-0.25)}{40}} = 0.06847\end{aligned}$$

Since both np and nq are more than 5 we can use the normal distribution as an approximation to the sampling distribution. At a significance level of 10%, the critical value for the test is – 1.28.

The standardized value of the sample proportion,

$$Z = \frac{\bar{p} - p_{H_0}}{\sigma_{\bar{p}}} = \frac{0.20 - 0.25}{0.06847} = -0.73$$



From above we can see that the standardized proportion falls in the acceptance region.

Hence the null hypothesis is accepted. It can be concluded at a significance level of 10%, that the percentage of women's clothing stores in Secunderabad which carry its products is not significantly less than the percentage in Hyderabad.

62. Answer : (e)

Reason : Standard error of mean for sample size 'm' = $\frac{\sigma}{\sqrt{m}} = 2.25 = A$

Standard error of mean for sample size 'n' = $\frac{\sigma}{\sqrt{n}} = 1.5 = B$

$$\frac{A}{B} = \frac{2.25}{1.5}$$

$$\text{or } \frac{\sigma/\sqrt{m}}{\sigma/\sqrt{n}} = 1.5$$

$$\text{or } \frac{\sigma}{\sqrt{m}} \times \frac{\sqrt{n}}{\sigma} = 1.5$$

$$\text{or } \sqrt{\frac{n}{m}} = 1.5$$

$$\text{or } n = 2.25m$$

Sample size = $m + n = m + 2.25m = 3.25m$.

$\therefore$ Standard error of mean for sample size 'm + n'

$$= \frac{\sigma}{\sqrt{m+n}} = \frac{\sigma}{\sqrt{3.25m}}$$

$$= \frac{1}{\sqrt{3.25}} \times \frac{\sigma}{\sqrt{m}}$$

$$= \frac{1}{\sqrt{3.25}} \times 2.25$$

$$= 1.2481$$

63. Answer : (d)

Reason : Let the following notations be used:

μ_1 = Population mean hours of relief provided by the drug of APL

μ_2 = Population mean hours of relief provided by the drug of OLL.

$$H_0 : \mu_1 = \mu_2$$

$$H_1 : \mu_1 > \mu_2$$

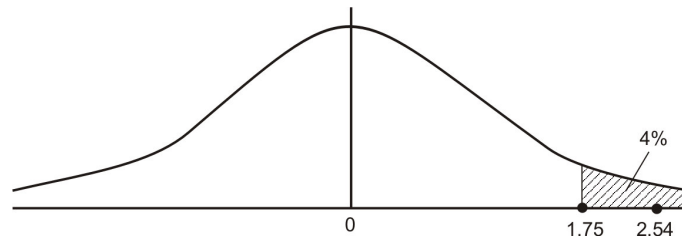
Estimated standard error of the difference between the two means

$$\begin{aligned}\hat{\sigma}_{\bar{x}_1 - \bar{x}_2} &= \sqrt{\frac{\hat{\sigma}_1^2}{n_1} + \frac{\hat{\sigma}_2^2}{n_2}} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \\ &= \sqrt{\frac{(1.5)^2}{80} + \frac{(2.5)^2}{64}} = 0.3547 \text{ hours}\end{aligned}$$

Standardization of difference between sample means

$$\begin{aligned}Z &= \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)_{H_0}}{\hat{\sigma}_{\bar{x}_1 - \bar{x}_2}} \\ &= \frac{(10.5 - 9.6) - 0}{0.3547} = 2.54\end{aligned}$$

Critical value at 4 percent level of significance = 1.75



So we can see that the difference between the sample means falls in the rejection region. Hence null hypothesis is rejected and alternative hypothesis is accepted.

∴ At a significance level of 4% we can conclude that the mean number of hours of relief provided by the drug produced by APL is significantly higher than OLL.

64. Answer : (d)

Reason : Let the following notations be used:

X : Number of years of experience

Y : Annual sales

$$\hat{Y} = a + bX$$

$$\Sigma X = 80 \quad \Sigma Y = 1060 \quad n = 10$$

$$\Sigma XY = 9048 \quad \Sigma X^2 = 782$$

$$\therefore b = \frac{n \Sigma XY - \Sigma X \Sigma Y}{n \Sigma X^2 - (\Sigma X)^2} = \frac{(10 \times 9048) - (80 \times 1060)}{(10 \times 782) - (80 \times 80)}$$

$$= \frac{5680}{1420} = 4.00$$

$$a = \bar{Y} - b\bar{X} = \left(\frac{1060}{10} \right) - 4 \left(\frac{80}{10} \right) = 74.00$$

∴ The regression relationship is : $\hat{Y} = 74 + 4X$

For number of years of experience (X) = 10,

Annual sales estimated ($\hat{Y}$) = $74 + 4(10) = 114$ (Rs in 000s)

∴ For 10 years of experience the estimated annual sales will be Rs.1,14,000.

65. Answer : (e)

Reason : Let the following notations be used:

X : Sales (Rs in lakhs)

Y : Operating costs (Rs in lakhs)

A	X	106	140	144	155	136	150	146	143	ΣX	= 1120
B	Y	100	114	120	125	105	124	122	118	ΣY	= 928
C	X - $\bar{X}$	-34	0	4	15	-4	10	6	3		
D	Y - $\bar{Y}$	-16	-2	4	9	-11	8	6	2		
E	(X - $\bar{X}$) ²	1156	0	16	225	16	100	36	9	Σ(X - $\bar{X}$) ²	= 1558
F	(Y - $\bar{Y}$) ²	256	4	16	81	121	64	36	4	Σ(Y - $\bar{Y}$) ²	= 582
G	C×D	544	0	16	135	44	80	36	6	Σ(X - $\bar{X}$) (Y - $\bar{Y}$)	= 861

$$\bar{X} = \frac{\sum X}{n} = \frac{1120}{8} = 140$$

$$\bar{Y} = \frac{\sum Y}{n} = \frac{928}{8} = 116$$

$$r = \frac{\sum (X - \bar{X})(Y - \bar{Y})}{\sqrt{\sum (X - \bar{X})^2 \sum (Y - \bar{Y})^2}}$$

$$= \frac{861}{\sqrt{1558 \times 582}} = 0.9042$$

66. Answer : (a)

Reason : $\hat{Y} = a + bX$

Given: $7 = a + b(2)$ (For X=2)

Or $7 = a + 2b$(A)

$11.8 = a + b(8)$ (For X=8)

Or $11.8 = a + 8b$(B)

$$\therefore b = \frac{(11.8 - 7)}{(8 - 2)} = 0.80$$

Putting b = 0.80 in (A)

$$7 = a + 2(0.80)$$

$$\text{or } a = 7 - 1.6 = 5.4$$

∴ The regression equation is

$$\hat{Y} = 5.4 + 0.80X$$

$$\text{Standard error of estimate; } S_e = \sqrt{\frac{\sum Y^2 - a \sum Y - b \sum XY}{n-2}}$$

$$\sum Y^2 = 791.31 \quad \sum XY = 452.7$$

$$\sum Y = 78.1 \quad n = 8$$

$$\begin{aligned} \therefore S_e &= \sqrt{\frac{791.31 - (5.4 \times 78.1) - (0.8 \times 452.7)}{8-2}} \\ &= 1.1113 \end{aligned}$$

67. Answer : (b)

$$\text{Reason : Coefficient of determination, } r^2 = 1 - \frac{\sum (Y - \hat{Y})^2}{\sum (Y - \bar{Y})^2}$$

Y denotes the observed values and $\hat{Y}$ denotes the estimated values.

$$\bar{Y} = \frac{\sum Y}{n} = \frac{792}{8} = 99$$

$$\sum (Y - \bar{Y})^2 = \sum Y^2 - n\bar{Y}^2 = 79144 - 8 \times 99^2 = 736$$

$$\therefore \text{Coefficient of determination, } r^2 = 1 - \frac{200}{736} = 0.7283$$

68. Answer : (e)

$$\begin{aligned} \text{Reason : Coefficient of correlation between X and Y} &= 0.85 \\ \therefore \text{Explained variation in Y} &= 0.85^2 = 0.7225 \\ \text{Unexplained variation in Y} &= 1 - 0.85^2 = 0.2775 \\ \text{Coefficient of correlation between Y and W} &= -0.92 \\ \therefore \text{Explained variation in Y} &= (-0.92)^2 = 0.8464 \\ \text{Unexplained variation in Y} &= 1 - (-0.92)^2 = 0.1536 \\ \text{Hence both (c) and (d) are correct.} \end{aligned}$$

69. Answer : (d)

Reason : Let the following notations be used:

X : Year

x : Coded value for year

Y : Sales

Year	$x = (X - \bar{X}) \times 2$	Y	xY	x^2
1997	$-2.5 \times 2 = -5$	35	-175	25
1998	$-1.5 \times 2 = -3$	60	-180	9
1999	$-0.5 \times 2 = -1$	85	-85	1
2000	$0.5 \times 2 = 1$	105	105	1
2001	$1.5 \times 2 = 3$	135	405	9
2002	$2.5 \times 2 = 5$	160	800	25
		580	870	$\sum x^2 = 70$

$$\begin{aligned}\bar{X} &= \frac{\Sigma X}{n} = 1999.5 \\ \hat{Y} &= a + bx \\ b &= \frac{\Sigma xY}{\Sigma x^2} = \frac{870}{70} = 12.43 \\ a = \bar{Y} &= \frac{\Sigma Y}{n} = \frac{580}{6} = 96.67\end{aligned}$$

∴ The linear trend equation is

$$\hat{Y} = 96.67 + 12.43x$$

The estimated sales for the year 2003:

$$X = 2003 \quad \therefore x = (2003 - 1999.5) \times 2 = 7$$

Estimated sales for the year 2003 = $96.67 + 12.43 \times 7 = \text{Rs.}183.68 \text{ lakh.}$

70. Answer : (b)

Reason : Let the linear trend equation be : $\hat{Y} = a + bx$

$x = X - 1998$ where $X = \text{time variable (year)}$

Given:

X	1999	2000	2001
$x = X - 1998$	1	2	3
Y	63.18	72.45	77.91
$\frac{Y - \hat{Y}}{\hat{Y}} \times 100$	8	5	
$\hat{Y}$	58.5	69	

$$1999: \quad \frac{Y - \hat{Y}}{\hat{Y}} \times 100 = 8$$

$$\text{or} \quad \frac{63.18 - \hat{Y}}{\hat{Y}} \times 100 = 8$$

$$\text{or} \quad \frac{63.18}{\hat{Y}} \times 100 = 108 \quad \text{or} \quad \hat{Y} = \frac{63.18}{108} \times 100 = \text{Rs.}58.5 \text{ lakh}$$

$$2000: \quad \frac{Y - \hat{Y}}{\hat{Y}} \times 100 = 5$$

$$\text{or} \quad \frac{72.45 - \hat{Y}}{\hat{Y}} \times 100 = 5$$

$$\text{or} \quad \frac{72.45}{\hat{Y}} \times 100 = 105 \quad \text{or} \quad \hat{Y} = \frac{72.45}{105} \times 100 = \text{Rs.}69 \text{ lakh.}$$

∴ From above we have

$$58.5 = a + b(1) \text{ i.e. } 58.5 = a + b \quad (A)$$

$$69 = a + b(2) \text{ i.e. } 69 = a + 2b \quad (B)$$

$$\therefore b = \frac{69-58.5}{2-1} = 10.5$$

Putting the value of b in (A) we get

$$a = 58.5 - 10.5 = 48$$

∴ The linear trend equation is: $\hat{Y} = 48 + 10.5 x$

Estimated value of Y for year 2001:

$$X = 2001 \quad \therefore x = 2001 - 1998 = 3$$

$$\hat{Y} = 48 + 10.5 (3) = 79.5$$

Relative cyclical residual for 2001:

$$\frac{Y - \hat{Y}}{\hat{Y}} \times 100 = \frac{77.91 - 79.5}{79.5} \times 100 = -2$$